

## MATHEMATICAL SIMULATION OF THE PROCESS OF DISPERSION OF CLAYS IN FILTRATION OF SOLUTIONS THROUGH SLIGHTLY PERMEABLE CLAY ROCKS

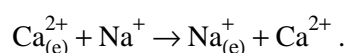
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*Based on concepts of the process of dispersion of clays, the author developed a model of the change in the filtration properties of slightly permeable clay rocks in filtration through them of solutions with a composition different from the initial composition. Of primary importance here are two processes, i.e., the porosity increase due to dispersion of a part of the porous collector and the increase in the viscosity of the resulting suspension. It is clear that these two processes oppositely affect the filtration factor. In the present work, the dynamics of the influence of these processes on the filtration factor is tracked and the exact analytical solution for the nonstationary problem in a one-dimensional case is obtained.*

In practice, one knows well the phenomenon of the increase in the permeability of slightly permeable clay rocks in filtration through them of highly mineralized solutions (as a rule, contaminated ones) compared to slightly mineralized or fresh solutions [1]. This effect and others that are similar in mechanism can be explained based on the concepts of the resulting structural rearrangement and subsequent dispersion of clay particles due to chemical reactions (ion exchange of the cations of a solution and the exchange complex of clay, reoxidation of iron ions with the subsequent change in the initial clay, etc.). A "new" clay is formed, whose particles are surrounded by a solution which carries an electric charge. Therefore, the motion of the medium causes the displacement of the particles of the "new" clay together with the flow. It is most simple to simulate this process as a gradual change of the clay to a suspension whose viscosity can be determined from theoretical and phenomenological considerations. The possibility of such processes occurring in a stratum is also confirmed by the practice of exploitation of slightly permeable clay-containing oil collectors: in the course of filtration, the composition of the passing water changes (in cations) and, in addition, the passing water frequently contains clay microparticles, which then form a layer of clay precipitate [2]. Two processes turn out to be substantial here for filtration: the increase in the porosity and consequently in the permeability due to dispersion and the increase in the viscosity of the resulting suspension. Below, we present the simplest model in which these concepts are realized.

**Model.** For definiteness, let us consider the reaction of replacement of the calcium cations of an exchange complex by sodium that is present in the incoming solution (a situation which occurs frequently in contamination of underground waters):



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Since here the structural rearrangement of the clay proceeds, the reaction can be considered to be irreversible. Then the system of equations for this type of process can be written as

$$\frac{\partial (mc)}{\partial t} + \nabla (\mathbf{V}c) + \frac{\partial N}{\partial t} = 0, \quad (1)$$

$$\frac{\partial m}{\partial t} + \nabla \mathbf{V} = 0, \quad (2)$$

$$\frac{\partial N}{\partial t} = \gamma \sqrt{E - N} c, \quad (3)$$

$$m - m_0 = \frac{\beta}{\rho} (N - N_0), \quad (4)$$

$$\mathbf{V} = - \frac{k(m)}{\mu(c)} \nabla p. \quad (5)$$

Equation (1) is the law of conservation of mass for  $\text{Na}^+$  ions of the filtered solution; Eq. (2) is the same for water; Eq. (3) reflects the exchange kinetics; Eq. (4) assigns the relationship between the porosity and the amount of sodium adsorbed; (5) is the filtration law of a fluid. The correction factor  $\beta$  accounts for the fact that the dispersing clay microparticle is a macroanion and, consequently, a certain finite number of elementary acts of cation exchange is needed for dispersion.

The system of equations (1)–(5) is rather complex, and we are unable to solve it in general form. However, all the physical features of the process can be tracked for a one-dimensional case with the condition of constancy of the total flow rate. Here we can obtain an exact analytical solution of system (1)–(5).

**Solution.** Let us consider a one-dimensional case ( $\nabla \rightarrow \partial/\partial x$ ) for  $V = \text{const}$ . We perform the replacement  $E - N = M$ . Then, expressing from Eq. (3)

$$c = - \frac{1}{\gamma \sqrt{M}} \frac{\partial M}{\partial t}, \quad (6)$$

it is possible to write (1) in a form that allows the first integral:

$$mc - \frac{2V}{\gamma} \frac{\partial \sqrt{M}}{\partial x} - M = \varphi(x). \quad (7)$$

The value is found from the conditions on the leading edge:  $M = M_0$  and  $m = m_0$ . It turns out that

$$\varphi(x) = -M_0. \quad (8)$$

Consequently, Eq. (7) has the form

$$\left( m_0 + \frac{M_0 - M}{\rho_e} \right) \frac{\partial M}{\partial t} + V \frac{\partial M}{\partial x} = \gamma \sqrt{M} (M_0 - M), \quad (9)$$

where we introduced the notation  $\rho_e = \rho/\beta$ . The characteristic system takes the form

$$\frac{dt}{m_0 + \frac{M_0 - M}{\rho_e}} = \frac{dx}{\gamma} = \frac{dM}{\gamma\sqrt{M}(M_0 - M)}. \quad (10)$$

The initial and boundary conditions are

$$t = 0: c = 0, M = M_0; \quad x = 0: c = c_1. \quad (11)$$

The solution is found for two stages. In the first stage (to the completion of the reaction at the point  $x = 0$ ), the solution is determined by conditions (11). Here

$$\frac{V}{\gamma\sqrt{M_0}} \ln \left[ \frac{\sqrt{M_0} + \sqrt{M}}{\sqrt{M_0} - \sqrt{M}} \right] - x = \frac{V}{\gamma\sqrt{M_0}} \ln \left[ \frac{4\sqrt{M_0}}{\gamma c_1 \left( t - \frac{m_0}{V} x \right)} - 1 \right], \quad (12)$$

$$c = \frac{4(M_0 - M)}{\left[ 4\sqrt{M_0 - \gamma c_1 \left( t - \frac{m_0}{V} x \right)} \right] \gamma \left( t - \frac{m_0}{V} x \right)}. \quad (13)$$

Relations (12) and (13) yield the form of the solution up to the moment  $T = 2\sqrt{M_0}/(\gamma c_1)$  (the completion of the reaction at the point  $x = 0$ ). After this, beginning from  $x = 0$ , a region is formed in which the reaction does not proceed. The boundary of this region moves; the velocity of its motion (of the trailing edge) is determined from the condition of balance for the calcium ions

$$\int_{x_2}^{x_1} m(c_1 - c) dx + \int_{x_2}^{x_1} M dx = M_0 x_1 \quad (14)$$

and appears to be equal to

$$V_b = \frac{V c_1}{M_0 + m_0 c_1}. \quad (15)$$

The solution for  $t > T$  is given for two regions (beginning from the leading edge) and is defined by relations (12) and (13) in the first region and by the condition  $M = 0$  for the trailing edge in the second region:

$$\frac{V}{\gamma\sqrt{M_0}} \ln \left[ \frac{\sqrt{M_0} + \sqrt{M}}{\sqrt{M_0} - \sqrt{M}} \right] - x = \frac{m_0 V_b}{\gamma\sqrt{M_0}} \ln \left[ \frac{\sqrt{M_0} + \sqrt{M}}{\sqrt{M_0} - \sqrt{M}} \right] - V_b(t - T), \quad (16)$$

$$c = \frac{V_b(M_0 - M)}{V - m_0 V_b}. \quad (17)$$

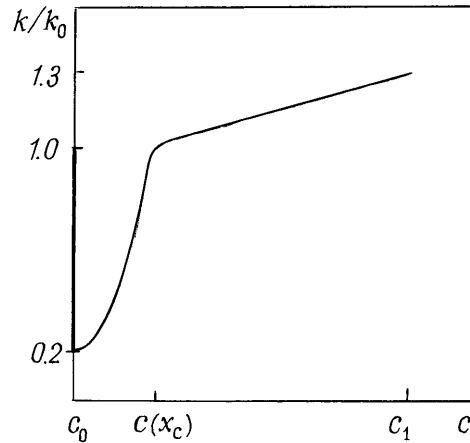


Fig. 1. Filtration factor vs. mineralization of the pore solution.

The point of contact of the two regions  $x_c$  moves according to the equation

$$t - \frac{m_0}{V} x_c = \frac{2\sqrt{M_0}}{\gamma c_1}. \quad (18)$$

Relation (18) follows from the continuity condition of the function  $M$ . It turns out that the concentration  $c$  at the point  $x_c$  is also continuous and continuously differentiable. It is of interest that the length of the region between the leading edge  $x_1 = Vt/m_0$  and the contact point  $l = x_1 - x_c$  is constant:

$$l = \frac{2V\sqrt{M_0}}{\gamma m_0 c_1}. \quad (19)$$

We will consider the process of dispersion of the clay particles in a generalized sense, i.e., either as the swelling or the dispersion proper of the clay. In both cases, a portion of the clay changes to a mobile state, i.e., it forms a suspension of a certain viscosity. The following two processes proceed simultaneously:

- 1) the increase in the porosity due to the transition of a fraction of the particles of the porous skeleton to a mobile state and its related increase in the penetration factor;
- 2) the increase in the viscosity of the resulting suspension with increase in the concentration of the suspension.

The porosity growth in the process is described by Eqs. (3) and (4). To relate the penetration factor to the porosity, we select the simple linear dependence

$$k(m) = Am, \quad A = \text{const}, \quad (20)$$

which in many cases agrees well with the field data [3].

In order to determine the dependence of the suspension viscosity on the concentration, we use the Mooney formula recommended in [4] and take the concentration of the suspension to be a multiple of the concentration of the calcium ions in the solution:

$$\mu(c) = \mu_0 \exp \left[ \frac{a(c_1 - c)}{1 - \delta(c_1 - c)} \right]; \quad \mu_0, a, \delta - \text{const}. \quad (21)$$

Using the solution (12)–(17) obtained, it is possible to plot, at an arbitrary point  $x$ , the filtration factor  $K = k/\mu$  versus the concentration of the solution. The form of the resulting dependence is given in Fig. 1.

The parameters  $a$ ,  $\delta$ , and  $\mu_0$  are chosen in such a way as to provide a fivefold increase in the initial value of the filtration factor. The curve given in Fig. 1 is in good agreement with the experiment ([1], p. 119).

It follows from Eq. (21) that when the concentration  $c$  reaches the value of  $(c_1 - 1/\delta)$  at the point  $x$ , we obtain the total cessation of filtration. This case where further filtration becomes impossible can be interpreted as limiting swelling.

Thus, the proposed mathematical model of variation in the filtration properties of slightly permeable clay rocks due to the dispersion of the clay component is based on the assumption of the interrelation between the processes of ion exchange (in particular, replacement of bivalent cations by univalent cations of the  $\text{Na}^+$ -type) and the subsequent dispersion of the clay particles. This assumption is confirmed by the experimental data [5]. The calculations performed by means of the model suggested also agree well with the experimental material (see Fig. 1).

## NOTATION

$m$ , porosity;  $c$ , concentration of sodium ions in the solution;  $N$ , concentration of sodium ions in the composition of the solid phase;  $\mathbf{V}$ , filtration velocity;  $t$ , time;  $E$ , exchange capacity;  $\gamma$ , reduced reaction constant;  $\rho$ , clay density;  $\beta$ , correction factor;  $p$ , pressure in the fluid;  $k$ , permeability;  $\mu$ , fluid viscosity. Subscripts: e, belongs to the exchange complex of clay; 0, initial state; b, trailing edge; c, contact point of two solutions.

## REFERENCES

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